
Sheet 11

1. Suppose we have a vector **a** in a three dimension space represented by three basis unit vectors v_1, v_2 and v_3 in the space as follows

$$a = [1 \;\; 2 \;\; 3]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Find the representation **b** of vector **a** using the three basis vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ related to the original three basis vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ as follows

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = M\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

2. Suppose we have a vector **a** in a frame represented by three basis unit vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ and the origin of the frame P_0 in homogenous coordinates as follows

$$a = [1 \;\; 2 \;\; 3 \;\; 0]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix}$$

Find the representation **b** of vector **a** using in a new frame characterized by the three basis vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ and $\mathbf{Q}_0$ related to the original frame as follows

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ Q_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix} = M\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix}$$

Note that the origin does not change for the two frames

3. Suppose we have a vector **a** in a frame represented by three basis unit vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ and the origin of the frame $\mathbf{P}_0$ in homogenous coordinates as follows

$$a = [1 \;\; 2 \;\; 3 \;\; 0]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix}$$

Find the representation **b** of vector **a** using in a new frame characterized by the three basis vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ and $\mathbf{Q}_0$ related to the original frame as follows

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ Q_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix} = M\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix}$$

Note that the origin of the second frame is related to the origin in the first frame with the following point vector addition in the original frame

$$Q_0 = v_1 + 2v_2 + 3v_3 + P_0$$

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4. Suppose we have a vector a in a three dimension space represented by three basis unit vectors v_1, v_2 and v_3 in the space as follows

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^T \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Find the representation **b** (as a transposed column vector) of vector **a** using the three basis vectors u_1, u_2, u_3 related to the original three basis vectors v_1, v_2 and v_3 as follows

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = M \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

5. Suppose we have a vector **a** in a frame represented by three basis unit vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ and the origin of the frame P_0 in homogenous coordinates as follows

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}^T \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix}$$

Find the representation **b** (as a transposed column vector)of vector **a** using in a new frame characterized by the three basis vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ and $\mathbf{Q}_0$ related to the original frame as follows

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ Q_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix} = M \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix}$$

Note that the origin does not change for the two frames

6. Suppose we have a vector **a** in a frame represented by three basis unit vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ and the origin of the frame $\mathbf{P}_0$ in homogenous coordinates as follows

$$a = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}^T \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix}$$

Find the representation **b** (as a transposed column vector)of vector **a** using in a new frame characterized by the three basis vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ and $\mathbf{Q}_0$ related to the original frame as follows

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ Q_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix} = M \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ P_0 \end{bmatrix}$$

Note that the origin of the second frame is related to the origin in the first frame with the following point vector addition in the original frame

$$Q_0 = v_1 + 2v_2 + 3v_3 + P_0$$

7. Try to summarize the difference between both the transformation matrix and the multiplying order in the two cases
   - When points and vectors data are organized as row vectors
   - When points and vectors data are organized as column vectors